

# Deuteron ground state properties and low energy P-N scattering $^1S_0$ and $^3S_1 - ^3D_1$ channels

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**Abstract.** Deuteron properties and P-N elastic scattering in  $^1S_0$  and  $^3S_1 - ^3D_1$  channels are described within the framework of a model containing non-local interactions that take into account quark exchange processes. Comparisons with other theoretical results are also made.

**PACS.** 13.75.Cs Nucleon-nucleon interactions (including antinucleons, deuterons, etc.) – 25.10.+s Nuclear reactions involving few-nucleon systems – 27.10.+h  $A \leq 5$

## 1 Introduction

Due to the high accuracy that was achieved in the experimental investigation of deuteron properties [1–3] and P-N scattering [4], it has become possible to compare theoretical predictions with experimental data and therefore to check different models of the nucleon-nucleon interaction. It is well known that at large separation distances the N-N potential is mainly determined by one-pion exchange processes while at short distances, nucleon structure should be taken into account properly. The correct solution of this problem can be obtained within the framework of a fundamental theory such as QCD. However, up to now, this program is far from complete. Therefore, a series of phenomenological potentials such as Paris, Bonn, Argonne and others containing a soft or hard repulsion at small distances have been used. These potentials nicely reproduce the scattering phase shifts. However, there are some discrepancies between the theoretical description of some deuteron ground state properties and the experimental results. If one looks at the correlations between quadrupole momentum  $Q$  and the asymptotic ratio of the S and D waves  $\eta = A_S/A_D$  [1], the theoretical results almost lie on a straight line for the different most popular N-N potentials. Concerning the experimental value of  $\eta$  the following remark could be done. In [5] the status of this problem is discussed. There are two different experimental methods to determine  $\eta$ . The first one is based on the pole extrapolation method and gives  $\eta = 0.0264 - 0.0275$ . These experimental data lie far from the theoretical line. The relativistic corrections and corrections due to two pion exchange processes [6] do not give any significant improvements. The second method deals with sub-Coulomb (d,p) reactions and gives  $\eta = 0.0256(4)$  [7]. This value is in bet-

ter agreement with the theoretical predictions. However, taking into account all the available experimental data one can see that this problem is still open at the present moment. Consequently, the nucleon structure should be considered more carefully [5].

Up to very recently there was the same problem in the triplet scattering length  $a_t$ -deuteron root-mean square radius  $r_D$  correlation [8]. However a recent analysis of the scattering data including Coulomb distortion [9] has shown the agreement between theoretical and experimental results removing this discrepancy.

To improve the agreement with experimental results, in [10,11] a deep local potential describing the P-N interaction has been proposed. In this case there is an additional spurious bound state that corresponds to a Pauli forbidden state (PFS). In this way, a good description of the deuteron ground state properties as well as scattering phase shifts have been obtained. In [8,12] another different way of describing the  $a_t - r_D$  correlation has been proposed. It consists in removing the discrepancy found with realistic local potentials by means of a phenomenologically added non-local contribution. In [8] the non-locality was considered only in the S-wave. The introduction of non-locality into the D-wave gives a much better description of the  $a_t - r_D$  and  $Q - \eta$  correlations [12]. However the physical motivation behind this approach should be clarified. The non-locality on NN force could also be found in [15–17]. A review of Moskow-type local potential and non-local NN interactions can be found in [13].

An alternative approximation, that will be used in the present paper, is the so-called quark cluster model. In it the nucleon is considered as a shell model cluster. To work out this approach, the Resonating Group Method (RGM)

[14] should be used. However, to avoid the technical difficulties encountered with this method, we have developed another model that is, actually, a simplified version of RGM. We will show that taking into account only the Pauli principle, the deuteron ground state properties as well as P-N scattering can be described. To do this, we will introduce a non-local interaction and show that the PFS, which are a consequence of the Pauli principle, are just the eigenfunctions of the effective inter-nucleon Hamiltonian with zero eigenvalue. In this paper we are interesting in the structure of the deuteron rather than in the general problem of P-N scattering. Therefore only  $^3S_1 - ^3D_1$  (that are the same as for D-problem) and  $^1S_0$  (that is added to complete the consideration) channels are taken into account.

Notice that the non-locality in the NN interaction can have other sources different from the Pauli principle. For instance, some commonly used potentials (as the Paris potential) have a  $\mathbf{p}^2$  dependence that represents a non-locality that may come from the effective character of the degrees of freedom used for describing the NN systems [15]. Minimal-relativity factors can also contribute to non-local effects [16]. Also a phenomenological non-local NN interaction which fits the NN data and produce the correct triton binding energy has been recently considered [17].

The paper is organized as follows. In the first part we will briefly describe the model and apply it to the P-N interaction. In the second part we will compare our results with those obtained with some potentials used for describing the deuteron and NN scattering. The summary and conclusions are laid out in the last part.

## 2 Description of the model

The most popular approach in the microscopic cluster theory is RGM. When it is applied to fermionic systems, the effective intercluster Hamiltonian contains non-local exchange terms. Due to technical difficulties, the application of this method is strongly restricted. In order to study the P-N problem in this way, one should introduce a non-relativistic quark-quark interaction that is not defined properly. Nevertheless, some calculations along these lines have been performed [18]. In order to solve these problems, some semi-phenomenological approaches to RGM could be used, for instance the Orthogonality Conditions Model [19]. This method gives the possibility of distinguishing the microscopic dynamics of the quarks by a proper choice of the intrinsic wave functions of the nucleons and the inter-nucleon semiphenomenological potential.

However, in this paper we propose a model that is another approximation to RGM. In it the P-N interaction is described by an effective Hamiltonian  $H$  that consists of an operator  $H^0$  that contains a local potential  $V^0$  and a non-local contribution  $W$  due to the exchange processes.

$$H = H^0 + W \quad (1)$$

In this sense we follow the RGM philosophy closely. The non-local contribution that describes the quark-exchange

processes is short range. Therefore, at large enough distances, the P-N interaction is mainly governed by the local potential  $V^0$  included in  $H^0$ .

Let us first discuss the direct potential. From the physical point of view,  $V^0$  should contain the main contribution from the one-pion exchange effects that are described by the one-pion exchange potential (OPEP). Therefore, in the triplet channel we use the OPEP as the local potential  $V^0$  written as

$$\begin{aligned} V^0 &= V_c(r) + V_t(r)\mathbf{S}_{12} \\ V_c(r) &= V_c^{OPEP}(r)f(\alpha r) \\ V_t(r) &= V_t^{OPEP}(r)f^3(\alpha r) \\ V_c^{OPEP}(r) &= V_0 \frac{e^{-\mu r}}{\mu r} \\ V_t^{OPEP}(r) &= V_0 \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r}, \end{aligned} \quad (2)$$

where  $\mathbf{S}_{12}$  is the tensor operator,  $f(\alpha, r) = 1 - e^{-\alpha r}$  is a cutoff function which regularises the singular potential and is taken as in [10].

The parameters of the OPEP have been taken from [10, 11]. The effective pion mass is defined as follows

$$\begin{aligned} \mu &= (2/3m_{\pi^{+,-}} + 1/3m_{\pi^0}) = 138.03 MeV \\ &= 0.6995 fm^{-1} \end{aligned} \quad (3)$$

The value of  $V_0$  is determined by the coupling constant  $f_{\pi NN}$  that is taken as  $f^2 = 0.07766$  and gives  $V_0 = -10.72 MeV$ . The cutoff parameter  $\alpha$  is taken as  $\alpha = 3.950832 fm^{-1}$ .

It is worth pointing out that the value of the constant  $f_{\pi NN}^2$  is under discussion at the present [20]. Arndt et al. [21] found the optimal value for the constant  $f_{\pi NN}^2$  to be 0.075. In turn Ericson et al. [22] used  $f_{\pi NN}^2 = 0.0808$ . We use in this work a constant  $f_{\pi NN}^2$  that lies between the values reported in [21] and [22].

At the same time, for a quantitative description of the P-N interaction, other meson exchange terms should be taken into account. It is well known that the central part of OPEP is too weak to reproduce the P-N interaction in a singlet channel where the tensor force gives no contribution. Therefore in the singlet channel we complement OPEP by a  $\sigma$ -meson exchange term. In [23] the value of  $m_\sigma$  is estimated as  $m_\sigma = 500 - 600 MeV$ . We approximate  $V_\sigma$  by a Yukawa term with the recoil correction:

$$V_\sigma = -\frac{g_s^2}{4\pi} m_\sigma \left(1 - \frac{m_\sigma^2}{4M^2}\right) \frac{e^{-m_\sigma r}}{m_\sigma r} f(\beta r), \quad (4)$$

where the cutoff factor  $f(\beta, r)$  is similar to  $f(\alpha, r)$  in (2.2).

The values of the parameters in (2.4) were taken to be:  $g_s^2/4\pi = 6.7$  (the value that is suggested in [23] is 7.1),  $m_\sigma = 500 MeV$ ,  $\beta = 3.51273 fm^{-1}$ .

The physical six-quark ( $6q$ ) many-particle wave function can be written as:

$$\tilde{\Psi}_{6q} = \mathbf{A} \tilde{\Psi}_{6q}, \quad (5)$$

where  $\mathbf{A}$  is the antisymmetrization operator while  $\tilde{\Psi}_{6q}$  is the arbitrary  $6q$  wave function. In the RGM ansatz  $\tilde{\Psi}_{6q} = \Psi_P \Psi_N \chi_{PN}$ , where  $\Psi_{P,(N)}$  is the internal wave function for protons or neutrons in terms of the intrinsic variables of the quarks;  $\chi_{PN}$  is a function that depends on the relative P-N distance  $\mathbf{r}$ , and is called the variational amplitude. In the shell model approach,  $\tilde{\Psi}_{6q}$  is simply the product of one-quark state wave functions. We define a Pauli-forbidden configuration  $\tilde{\Psi}_{6q} = \tilde{\Psi}_{PFS}$  ( $\tilde{\Psi}_{6q} \neq 0$ ) if the corresponding  $\Psi_{6q} = 0$ . The relative motion vector  $\Phi_{PFS} = \langle \Psi_P \Psi_N | \tilde{\Psi}_{PFS} \rangle_r$ , where the integration is taken over the intrinsic variables of P and N except  $\mathbf{r}$ , is just the PFS. In the RGM ansatz  $\Phi_{PFS} = \chi_{PN,PFS}$ . To take into account the PFS, one can orthogonalize it to the physical relative motion wave function. This can be done in various ways. For instance, one can apply the Orthogonality Condition Model of Saito [19]. In the model with a deep local potential [10,11], the PFS is just the spurious bound state.

In our model we embed the PFS within the spectrum of the effective Hamiltonian with zero eigenvalue. First of all, to describe the P-N interaction, we consider the  $^3S_1 - ^3D_1$  and  $^1S_0$  channels with a given total angular momentum  $J = 1$ . In the triplet channel there is a bound state that corresponds to the deuteron ground state. One can define the radial relative motion PFS vector with given S,L and J as follows:

$$\phi_{PFS}^{J,L,S} = \langle [ [\Psi_N \Psi_P]_S Y_L ]_{J,L,S} | \tilde{\Psi}_{PFS} \rangle_r, \quad (6)$$

where the bracket  $[\ ]$  means vector coupling and  $Y$  is the spherical function of the relative P-N channel with a given orbital angular momentum  $L$ . The integration in (2.6) is taken over all intrinsic variables of P and N and over the angular variables of  $\mathbf{r}$  but not the radial variable  $r$ . To calculate the PFS we consider the harmonic oscillator shell model for quarks inside the nucleon. In the singlet channel the PFS is written as

$$\phi_{PFS}^{J=0,L=0,S=0} = u_{n=0,L=0}(\nu, r), \quad (7)$$

where  $u_{n,L}(\nu, r)$  is the normalized radial harmonic oscillator wave function with the radial quantum number  $n$  and angular momentum  $L$ , while  $\nu$  is the frequency parameter defined as  $\nu = \mu\omega/\hbar$ . To take into account the non-locality in the triplet channel, we choose the PFS as follows

$$\phi_{PFS}^{J=1,L,S=1} = a_L u_{n=0,L}(\nu, r), \quad (8)$$

where  $L = 0, 2$  and the coefficients  $a_L$  are normalized by the condition

$$|a_0|^2 + |a_2|^2 = 1. \quad (9)$$

To choose the value of  $\nu$  within the framework of the RGM one can try to use it to fit the nucleon radius. However, the shell model is too crude to describe the nucleon structure. It is known that exclusion of the PFS leads to the appearance of an almost energy-independent node in the intercluster relative motion wave function that is called the "structural" core. The position of this node  $r_c$  corresponds to the position of the effective NN repulsion in local

potential models. The value of  $r_c$  is determined by the oscillator frequency parameter  $\nu$ . Therefore, we will consider  $\nu$  as the parameter that reproduces reasonable values of the repulsive core and nucleon radius at the same time.

At this moment we would like to discuss one interesting possibility. From a physical point of view, the position of the repulsive core in the NN channel (identified with the structural core) can slightly depend on the relative energy and may decrease when the energy increases. This effect can be obtained by increasing the oscillator frequency parameter  $\nu$ . As a consequence the size of the nucleon decreases too. If one takes into account the fact that the Compton wave length of the particle decreases at high energy, the trick of using a parameter  $\nu$  that increases slightly when the energy is increased seems reasonable. On the other hand, this gives the possibility of obtaining a better description of scattering data as will be shown later.

In the partial wave representation, the Schrödinger equation will be written as a system of coupled equations:

$$\sum_{L'} (H_{L,L',S}^0 + W_{L,L',S}) \psi^{J,L',S} = E \psi^{J,L,S}. \quad (10)$$

In the triplet channel  $S = 1$  with  $J = 1$ , the orbital momentum  $L$  runs over  $L = 0, 2$ , while in the singlet  $S = 0$  channel with  $J = 0$  only one equation with  $L = 0$  survives. The local part  $H_{L,L',S}^0$  is calculated using the local potential (2.2) plus (2.4) in the  $S = 0$  channel.

In our approach the non-local part  $W_{L,L',S}$  is written using  $H^0$  and the PFS given by (2.7) and (2.8) :

$$W_{L,L',S} = \sum_{L_1 L_2} H_{L,L_1,S}^0 | \phi_{PFS}^{J,L_1,S} \rangle \lambda_S \langle \phi_{PFS}^{J,L_2,S} | H_{L_2,L',S}^0, \quad (11)$$

where the coefficients  $\lambda_S$  are chosen as:

$$\lambda_S = - \left[ \sum_{L,L'} \langle \phi_{PFS}^{J,L,S} | H_{L,L',S}^0 | \phi_{PFS}^{J,L',S} \rangle \right]^{-1}. \quad (12)$$

In this model for the intercluster Hamiltonian  $H$  it is easy to check that the wave functions corresponding to PFS  $\phi_{PFS}^{J,L,S}$  given by (2.7) and (2.8) are orthogonal to the corresponding physical wave function  $\psi^{J,L,S}$ , which is a solution of (2.10)

$$\sum_L \langle \phi_{PFS}^{J,L,S} | \psi^{J,L,S} \rangle = 0, \quad (13)$$

where again  $L = 0$  for the singlet channel and  $L = 0, 2$  for the triplet.

Moreover, one can see that the PFS, which are equal to  $\phi_{PFS}^{J=0,L=0,S=0}$  in the singlet channel and to the row  $\left( \begin{array}{c} \phi_{PFS}^{J=1,L=0,S=1} \\ \phi_{PFS}^{J=1,L=2,S=1} \end{array} \right)$  in the triplet channel, are just the eigenvectors of the Hamiltonian  $H$  with zero eigenvalues.

**Table 1.** Deuteron ground state properties obtained using different models are compared with experimental data. The results of calculations with the Paris, Argonne, Reid soft core potentials and Moscow potential are taken from [10]. The results of the present paper are labelled as Model. The results reported in [8,12] are also given

	Paris*	Argonne*	RHC*	[8,12]	Moscow	Model	Exp.
$E(MeV)$	-2.225	-2.225	-2.2246	-2.2242	-2.2245	-2.2245890	-2.2245890(2) <sup>[30]</sup>
$r_D(fm)$	1.9716		1.9569	1.953	1.9592	1.965*	1.966(13) <sup>[9,40]**</sup> 1.971 <sup>[40]</sup> 1.97535(85) <sup>[41]</sup>
$\eta$	0.0261	0.0265	0.0262	0.0287	0.0269	0.02710	0.0271(4) <sup>[32]</sup> 0.0252(1) <sup>[42]</sup> 0.0256(4) <sup>[7]</sup>
$A_S(fm^{-1})$	0.8869		0.8776	0.8898	0.8814	0.8836	0.8838(4) <sup>[31]</sup> 0.8846(8) <sup>[33]</sup>
$Q(fm^2)$	0.2789	0.286	0.2796	0.2862	0.285*	0.2859*	0.2859(3) <sup>[34]</sup>
$\mu_D(n.m.)$					0.854*	0.8580*	0.857406(1) <sup>[35]</sup>
$P_D$	5.77	6.13	6.497	6.544	6.75	5.86	6.0(2.0) <sup>[36]</sup> 5.0(2.0) <sup>[32]</sup>
$r_t(fm)$	1.765	1.81		1.724	1.745	1.756	1.754(8) <sup>[29]</sup>
$f_{\pi NN}^2$	0.078	0.081	0.0757		0.07745	0.07766	0.0776(9) <sup>[37]</sup> 0.075 <sup>[42]</sup>
$a_t(fm)$	5.427	5.46	5.39	5.418	5.413	5.413	5.419(7) <sup>[29]</sup>

\* the notation in [29]

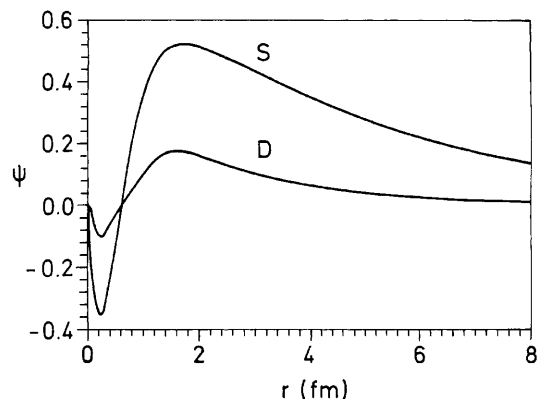
\* Means that relativistic corrections [24] are taken into account

\*\* The deuteron point radius is taken from [40] where it is obtained from the charge radius given in [9]

### 3 Results

First of all, we will consider the ground state properties of the deuteron which correspond to the triplet channel. In this approach the local potential in the triplet channel coincides with the OPEP and in our model there are three free parameters to be adjusted to the experimental data: the oscillator frequency parameter  $\nu$ , the cutoff parameter  $\alpha$  and the admixture of the D- component in the PFS  $a_2$ . To satisfy the unitarity condition of the S- matrix one should use real values for the coefficients of  $a_L$ . The values  $\nu = \nu_D = 4.7 fm^{-2}$  and  $a_2^2 = 0.22$  are used. To justify our choice one can look for the S- and D-wave functions that are displayed in Fig. 1. In our case the nodes in S- and D-waves coincide and this picture corresponds to a local potential with a hard repulsion at  $r_C \approx 0.6 fm$  which is determined by  $\nu$  and does not depend on angular momentum  $L$ . On the other hand, the radius of the nucleon (within the harmonic shell model without Coulomb effects) calculated with this value of  $\nu$  is  $0.57 fm$ .

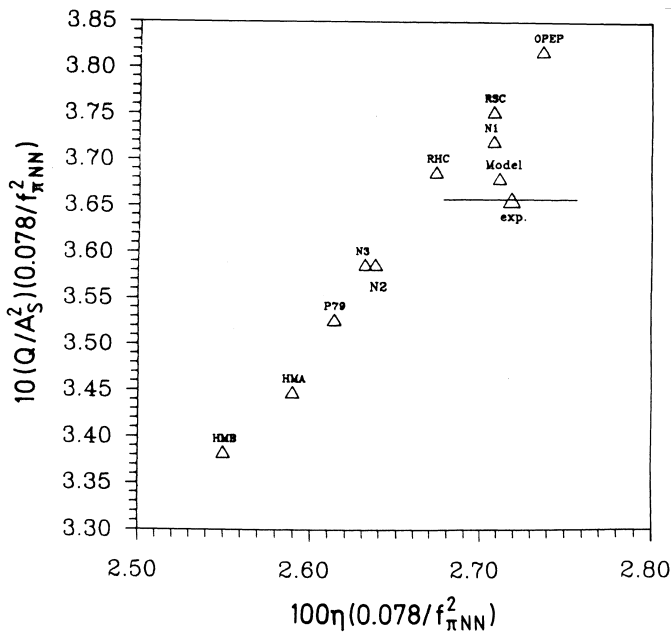
Table 1 displays the results of our calculations together with the experimental data and the results obtained using the phenomenological Paris, Argonne and Reid soft-core potentials reported in [10]. The results given in [8, 12] obtained with a phenomenological non-local part as well as the results obtained with the deep local potential of [11] (labelled as Moscow) are also presented. In this table the following quantities of the deuteron are presented: the binding energy of the deuteron  $E$ , the root mean square radius  $r_D$ , the asymptotic ratio of the S and D waves  $\eta = A_S/A_D$ , the quadrupole momentum  $Q$ , the magnetic momentum  $\mu_D$ , the admixture of the D-wave



**Fig. 1.** S - and D - deuteron wave functions obtained using our model

$P_D$ , the effective P-N range in the triplet channel  $r_t$  and the triplet scattering length  $a_t$ . For the sake of completeness the value of the coupling constant  $f_{\pi NN}^2$  used by the different potentials is also given.

The relativistic corrections to  $r_D$ ,  $Q$  and  $\mu_D$  have been calculated according to [24] (see also [25,26]). The corrections to  $r_D$  are rather small. The value of  $Q$  decreases by an amount of  $1.8 \cdot 10^{-3} fm^2$  when the relativistic corrections have been taken into account in all the considered calculations, while the  $\mu_D$  correction depends significantly on the model used. Our data for  $Q$  and  $r_D$  reported in Table 1 do not contain the exchange meson currents (EMC) that are corrections of  $1/M^3$  order. However, it was realized that they could give significant contributions [27].



**Fig. 2.**  $\frac{Q}{A_s^2}$  as a function of  $\eta$  for different realistic local potentials together with the experimental data. The results of our model are presented by the cross. The  $Q$  as well as  $\eta$  are reduced according to the coupling constant  $\pi NN$  [1]

In the potential models (like the present one) the pionic degrees of freedom are not taken into account explicitly and consequently, the EMC corrections have to be calculated perturbatively. However, the result strongly depends on the type of  $\pi NN$  interaction. We have estimated these EMC corrections using the masses and coupling constants given in [27]. For the quadrupole moment they could vary from 0.001 to 0.003  $fm^2$  for the pseudoscalar coupling and from 0.004 to 0.006  $fm^2$  when the pseudovector coupling is used. Realizing the significance of the EMC corrections, we do not consider them in present paper and leave this problem to be investigated in detail in the next future.

In view of the simplicity of our model, there is no striking discrepancies between the results obtained with it and the experimental values as it can be seen from Table 1.

In Fig. 2 the  $Q-\eta$  correlation for the different models is displayed. This figure has been taken from [1] and we have added our result. One can see that the value obtained in our model agrees with the experimental data. However, if the EMC correction to  $Q$  is added, our value is shifted up around of 3 % lying on the straightline of the theoretical potentials.

There are other very important quantities that give some insight into the inner structure of the deuteron. The structure functions  $A(q^2)$  and  $B(q^2)$  that determine the elastic scattering on an unpolarized deuteron are given by

$$\begin{aligned} A(q^2) &= F_C^2(q^2) + \frac{8}{9}\gamma F_Q(q^2) + \frac{2}{3}F_M^2(q^2) \\ B(q^2) &= \frac{4}{3}\gamma(\gamma + 1)F_M^2(q^2) \end{aligned} \quad (14)$$

, where  $\gamma = q^2/4M_D^2$  while  $F_S, F_Q$  and  $F_M$  are the charge, quadrupole and magnetic form factors of deuteron respectively. They are determined by the P-N relative motion wave function in the deuteron and by the inner structure of nucleons. For the magnetic  $G_M$  and electric  $G_E$  form-factors of nucleons we use the following parameterization [28]:

$$\frac{G_M^P(q^2)}{\mu_P} = \frac{G_M^N(q^2)}{\mu_N} = g_M(q^2)f(q^2) \quad (15)$$

and

$$\begin{aligned} G_E^P(q^2) &= \left[ \left(1 + \frac{q^2}{2M}\right) \frac{1 + 3q^2/2M^2}{1 + q^2/2M^2} g_E(q^2) \right. \\ &\quad \left. - \frac{3q^2}{4M^2} g_M(q^2) \right] f(q^2) \end{aligned} \quad (16)$$

where

$$\begin{aligned} g_E(q^2) &= \frac{(1 + 3q^2/4M^2)(1 + q^2/2M^2)}{(1 + q^2/4M^2)(1 + 3q^2/2M^2)} g_M(q^2) \\ g_M(q^2) &= 1 \end{aligned} \quad (17)$$

and

$$f(q^2) = \frac{1}{(1 + q^2/2M)^2} \exp \left[ -\frac{1}{2a} \left( \frac{q^2}{1 + q^2/2M} \right) \right] \quad (18)$$

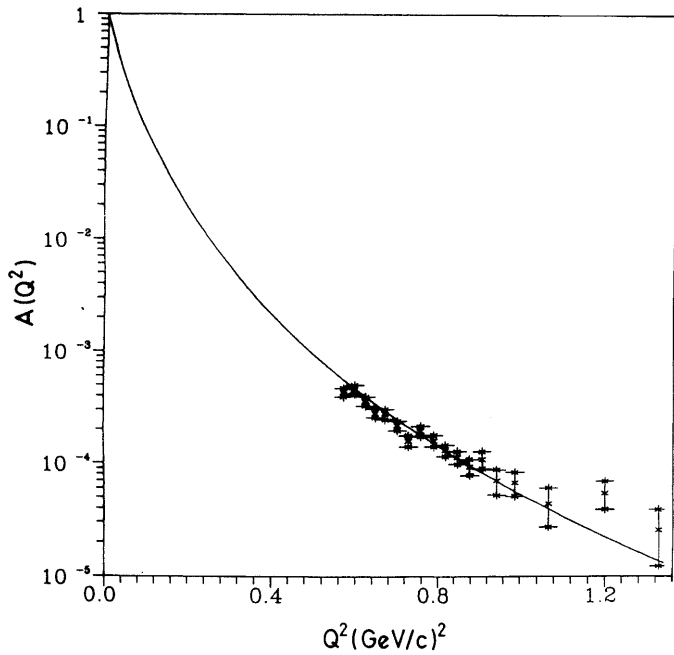
The value of the parameters are  $a = 0.3GeV/c^2$  and  $M = 0.93828GeV$ .

The results of calculations of  $A(q^2)$  and  $B(q^2)$  in our model are displayed in Fig. 3 and Fig. 4. One can see that within the framework of the nonrelativistic approach, reasonably good results are obtained up to  $q^2 = 1(GeV/c)^2$ .

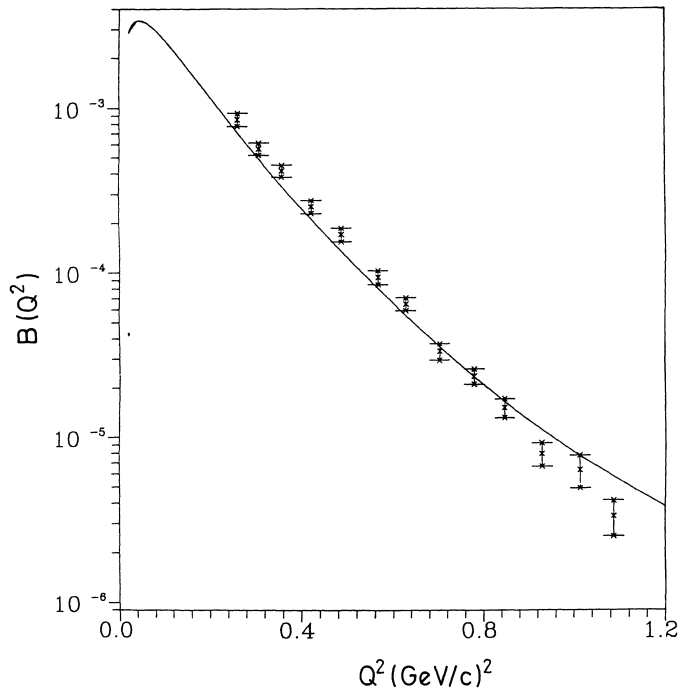
The S and D wave phase shifts in the triplet channel are presented in Fig. 5. The dashed line has been obtained when the parameters of our model ( $\nu$  and  $a_2$ ) have been fitted to describe the deuteron properties reported previously. As pointed out before, the use of an oscillator frequency depending on energy seems reasonable and can be useful. Therefore, we take the following energy-dependence:

$$\nu(E) = \nu + A_{T(S)}E, \quad (19)$$

where  $\nu$  is the frequency parameter of the deuteron used previously,  $A_T = 4.95 \cdot 10^{-3} fm^{-2} MeV^{-1}$ ,  $A_S = 3 \cdot 10^{-4} fm^{-2} MeV^{-1}$  and the  $T(S)$  indexes refer to triplet and singlet channels respectively. The results of these calculations are displayed in Fig. 5 and 6 by solid lines. In the singlet channel case no difference is practically found in the phase-shifts by using  $\nu(E)$  or simply  $\nu$  up to 350  $MeV$ . One can see that the S-wave phase shifts are nicely reproduced by our model. On the other hand, the fit of the D-wave phase shifts is not so good. It can be understood if one takes into account the fact that in our simple approach no spin-orbit interaction has been explicitly dealt with.



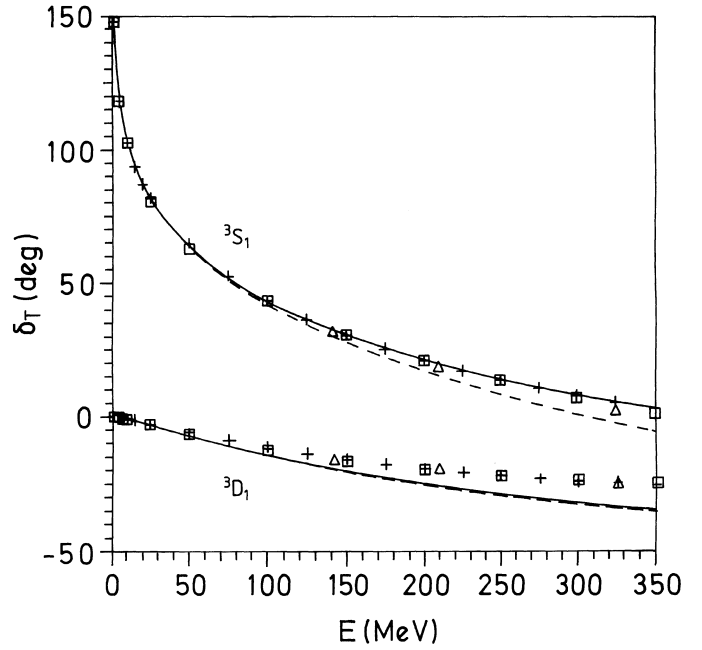
**Fig. 3.** Deuteron form factor  $A(q^2)$  (see text for explanation). The experimental data come from [43]



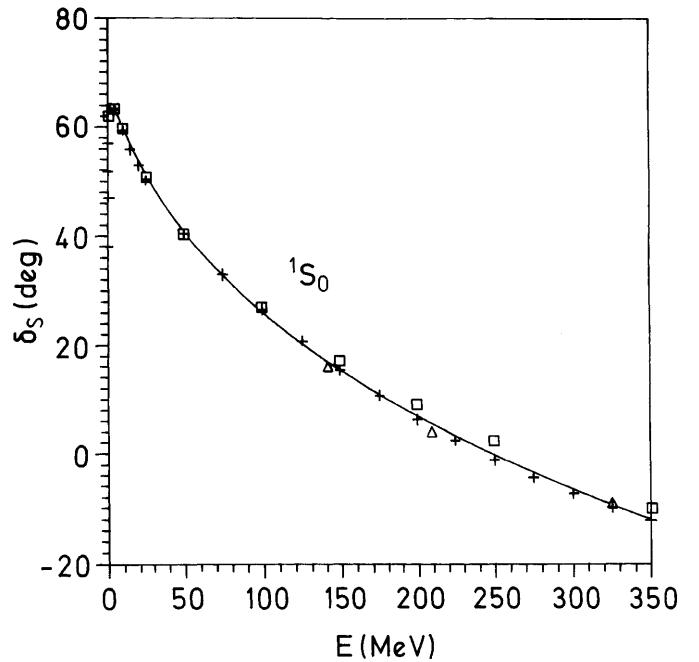
**Fig. 4.** Deuteron form factor  $B(q^2)$  (see text for explanation). The experimental data come from [44]

## 4 Summary

In this paper a model of intercluster dynamics with a non-local interaction is applied to the problem of P-N interaction. This model is an approximation to RGM. In it, the microscopic quark dynamics is hidden in the oscillator frequency and in the local part of the P-N interaction.



**Fig. 5.** The phase shifts in  $^3S_1$  and  $^3D_1$  channels obtained in our model. The solid lines correspond to  $A_T = 4.95 \times 10^{-3} fm^{-2} MeV^{-1}$  in (3.6) while the dashed lines correspond to  $A_T = 0 fm^{-2} MeV^{-1}$ . The experimental data come from [4](+), [38]( $\Delta$ ) and [39]( $\square$ )



**Fig. 6.** The phase shifts in  $^1S_0$  channels obtained in our model with  $A_S = 3 \times 10^{-4} fm^{-2} MeV^{-1}$  in (3.6). The experimental data come from [4](+), [38]( $\Delta$ ) and [39]( $\square$ )

The deuteron ground state properties as well as the S-wave scattering phase shifts up to 350 MeV are well reproduced with this approach. The D-wave phase-shifts for large values of the energy are not reproduced so well for the reasons

pointed out at the end of Section III. Our model contains three free parameters in the triplet channel, namely the cutoff  $\alpha$  of the OPEP potential, the oscillator frequency  $\nu$  and the S-D wave admixture in the PFS  $a_2$ . In the singlet channel the model also has three parameters: the cutoffs  $\alpha$  and  $\beta$  of the OPEP and  $\sigma$ -meson exchange potentials and the oscillator frequency  $\nu$ . In the phase-shift analysis one parameter more  $A_{T(S)}$  was used.

In order to compare this model with the other approximations to the RGM reported in the text, the following remarks should be made : a) The PFS in our model are embedded within the spectrum of the effective Hamiltonian as was done in [10,11]. However, in our approach the energy of the PFS is fixed and simply set equal to zero, which reduces the number of parameters. b) The non-local character of the P-N interaction in our model is similar to that of [8,12]. However, the non-local part is built up in a regular way.

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